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SOLVED PROBLEMS 1. A machine of mass 75 kg is mounted on springs and is fitted with a dashpot to damp out vibrations. There are three springs each of stiffness 10 N/mm and it is found that the amplitude of vibration diminishes from 38.4 mm to 6.4 mm in two complete oscillations. Assuming that the damping force varies as the velocity, determine : 1. the resistance of the dash-pot at unit velocity ; 2. the ratio of the frequency of the damped vibration to the frequency of the undamped vibration ; and 3. the periodic time of the damped vibration. Solution. Given : m = 75 kg ; s = 10 N/mm = 10 ×103 N/m ; x1 = 38.4 mm = 0.0384 m ; x3 = 6.4 mm = 0.0064 m Since the stiffness of each spring is 10 × 103 N/m and there are 3 springs, therefore total stiffness, 1. Resistance of the dashpot at unit velocity 2. Ratio of the frequency of the damped vibration to the frequency of undamped vibration 2. The mass of a single degree damped vibrating system is 7.5 kg and makes 24 free oscillations in 14 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Determine : 1. stiffness of the spring, 2. logarithmic decrement, and 3. damping factor, i.e. the ratio of the system damping to critical damping. Solution. Given : m = 7.5 kg Since 24 oscillations are made in 14 seconds, therefore frequency of free vibrations, 3(i) The measurements on a mechanical vibrating system show that it has a mass of 8 kg and that the springs can be combined to give an equivalent spring of stiffness 5.4 N/mm. If the vibrating system have a dashpot attached which exerts a force of 40 N when the mass has a velocity of 1 m/s, find : 1. critical damping coefficient, 2. damping factor, 3. logarithmic decrement, and 4. ratio of two consecutive amplitudes. Solution. Given : m = 8 kg ; s = 5.4 N/mm = 5400 N/m Since the force exerted by dashpot is 40 N, and the mass has a velocity of 1 m/s , therefore Damping coefficient (actual), 3 (ii) An instrument vibrates with a frequency of 1 Hz when there is no damping. When the damping is provided, the frequency of damped vibrations was observed to be 0.9 Hz. Find 1. the damping factor, and 2. logarithmic decrement. Solution. Given : fn = 1 Hz ; fd = 0.9 Hz 1. Damping factor Let m = Mass of the instrument in kg, c = Damping coefficient or damping force per unit cc= velocity in N/m/s, and Critical damping coefficient in N/m/s. We know that natural circular frequency of undamped vibrations, 4(i) A coil of spring stiffness 4 N/mm supports vertically a mass of 20 kg at the free end. The motion is resisted by the oil dashpot. It is found that the amplitude at the beginning of the fourth cycle is 0.8 times the amplitude of the previous vibration. Determine the damping force per unit velocity. Also find the ratio of the frequency of damped and undamped vibrations. Solution. Given : s = 4 N/mm = 4000 N/m ; m = 20 kg Damping force per unit velocity Let c = Damping force in newtons per unit velocity i.e. in N/m/s xn = Amplitude at the beginning of the third cycle, xn+1 = Amplitude at the beginning of the fourth cycle = 0.8 xn We know that natural circular frequency of motion, 4(ii) Derive an expression for the natural frequency of single degrees of freedom system. We know that the kinetic energy is due to the motion of the body and the potential energy is with respect to a certain datum position which is equal to the amount of work required to move the body from the datum position. In the case of vibrations, the datum position is the mean or equilibrium position at which the potential energy of the body or the system is zero. In the free vibrations, no energy is transferred to the system or from the system. Therefore the summation of kinetic energy and potential energy must be a constant quantity which is same at all the times. In other words, 5. A vertical shaft of 5 mm diameter is 200 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation and the maximum bending stress when the shaft is rotating at 75% of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. E = 200 GN/m2. Solution. Given : d = 5 mm = 0.005 m ; l = 200 mm = 0.2 m ; m = 50 kg ; e = 0.25 mm = 0.25 × 10–3 m ; E = 200 GN/m2 = 200 × 109 N/m2 Critical speed of rotation We know that moment of inertia of the shaft, Since the shaft is supported in long bearings, it is assumed to be fixed at both ends. We know that the static deflection at the centre of the shaft due to a mass of 50 kg, Maximum bending stress Let σ = Maximum bending stress in N/m2, and N = Speed of the shaft = 75% of critical speed = 0.75 Nc . . . (Given) When the shaft starts rotating, the additional dynamic load (W1) to which the shaft is subjected, may be obtained by using the bending equation, We know that for a shaft fixed at both ends and carrying a point load (W1) at the centre, the maximum bending moment 6.(i) A shaft 50 mm diameter and 3 metres long is simply supported at the ends and carries three loads of 1000 N, 1500 N and 750 N at 1 m, 2 m and 2.5 m from the left support. The Young's modulus for shaft material is 200 GN/m2. Find the frequency of transverse vibration. Solution. Given : d = 50 mm = 0.05 m ; l = 3 m, W1 = 1000 N ; W2 = 1500 N ; W3 = 750 N; E = 200 GN/m2 = 200 × 109 N/m2 The shaft carrying the loads is shown in Fig. 23.13 We know that moment of inertia of the shaft, 6.(ii) Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid-point. The density of the shaft material is 40 Mg/m3, and Young's modulus is 200 GN/m2. Assume the shaft to be freely supported. TUTORIAL PROBLEMS 1. A beam of length 10 m carries two loads of mass 200 kg at distances of 3 m from each end together with a central load of mass 1000 kg. Calculate the frequency of transverse vibrations. Neglect the mass of the beam and take I = 109 mm4 and E = 205×10 3 N/mm2. [Ans. 13.8 Hz] 13.8 2. A vertical shaft 25 mm diameter and 0.75 m long is mounted in long bearings and carries a pulley of mass 10 kg midway between the bearings. The centre of pulley is 0.5 mm from the axis of the shaft. Find (a) the whirling speed, and (b) the bending stress in the shaft, when it is rotating at 1700 r.p.m. Neglect the mass of the shaft and E = 200 GN/m2. [Ans. 3996 r.p.m ; 12.1 MN/m2] 3. A shaft of 100 mm diameter and 1 metre long is fixed at one end and the other end carries a flywheel of mass 1 tonne. The radius of gyration of the flywheel is 0.5 m. Find the frequency of torsional vibrations, if the modulus of rigidity for the shaft material is 80 GN/m2. [Ans. 8.9 Hz] 4.The two rotors A and B are attached to the end of a shaft 500 mm long. The mass of the rotor A is 300 kg and its radius of gyration is 300 mm. The corresponding values of the rotor B are 500 kg and 450 mm respectively. The shaft is 70 mm in diameter for the first 250 mm ; 120 mm for the next 70 mm and 100 mm diameter for the remaining length. The modulus of rigidity for the shaft material is 80 GN/m2. Find : 1. The position of the node, and 2. The frequency of torsional vibration. [Ans. 225 mm from A ; 27.3 Hz] 5.4 Forced vibration of damped, single degree of freedom, linear spring mass systems. Finally, we solve the most important vibration problems of all. In engineering practice, we are almost invariably interested in predicting the response of a structure or mechanical system to external forcing. For example, we may need to predict the response of a bridge or tall building to wind loading, earthquakes, or ground vibrations due to traffic. Another typical problem you are likely to encounter is to isolate a sensitive system from vibrations. For example, the suspension of your car is designed to isolate a sensitive system (you) from bumps in the road. Electron microscopes are another example of sensitive instruments that must be isolated from vibrations. Electron microscopes are designed to resolve features a few nanometers in size. If the specimen vibrates with amplitude of only a few nanometers, it will be impossible to see! Great care is taken to isolate this kind of instrument from vibrations. That is one reason they are almost always in the basement of a building: the basement vibrates much less than the floors above. We will again use a spring-mass system as a model of a real engineering system. As before, the spring-mass system can be thought of as representing a single mode of vibration in a real system, whose natural frequency and damping coefficient coincide with that of our spring-mass system. We will consider three types of forcing applied to the spring-mass system, as shown below: External Forcing models the behavior of a system which has a time varying force acting on it. An example might be an offshore structure subjected to wave loading. Base Excitation models the behavior of a vibration isolation system. The base of the spring is given a prescribed motion, causing the mass to vibrate. This system can be used to model a vehicle suspension system, or the earthquake response of a structure. Rotor Excitation models the effect of a rotating machine mounted on a flexible floor. The crank with small mass rotates at constant angular velocity, causing the mass m to vibrate. Of course, vibrating systems can be excited in other ways as well, but the equations of motion will always reduce to one of the three cases we consider here. Notice that in each case, we will restrict our analysis to harmonic excitation. For example, the external force applied to the first system is given by The force varies harmonically, with amplitude and frequency . Similarly, the base motion for the second system is and the distance between the small mass and the large mass m for the third system has the same form. We assume that at time t=0, the initial position and velocity of each system is In each case, we wish to calculate the displacement of the mass x from its static equilibrium configuration, as a function of time t. It is of particular interest to determine the influence of forcing amplitude and frequency on the motion of the mass. We follow the same approach to analyze each system: we set up, and solve the equation of motion. 5.4.1 Equations of Motion for Forced Spring Mass Systems Equation of Motion for External Forcing We have no problem setting up and solving equations of motion by now. First draw a free body diagram for the system, as show on the right Newton's law of motion gives Rearrange and susbstitute for F(t) Check out our list of solutions to standard ODEs. We find that if we set , our equation can be reduced to the form which is on the list. The (horrible) solution to this equation is given in the list of solutions. We will discuss the solution later, after we have analyzed the other two systems. Equation of Motion for Base Excitation Exactly the same approach works for this system. The free body diagram is shown in the figure. Note that the force in the spring is now k(x-y) because the length of the spring is . Similarly, the rate of change of length of the dashpot is d(x-y)/dt. Newton's second law then tells us that Make the following substitutions and the equation reduces to the standard form Given the initial conditions and the base motion we can look up the solution in our handy list of solutions to ODEs. Equation of motion for Rotor Excitation Finally, we will derive the equation of motion for the third case. Free body diagrams are shown below for both the rotor and the mass Note that the horizontal acceleration of the mass is Hence, applying Newton's second law in the horizontal direction for both masses: Add these two equations to eliminate H and rearrange To arrange this into standard form, make the following substitutions whereupon the equation of motion reduces to Finally, look at the picture to convince yourself that if the crank rotates with angular velocity , then where is the length of the crank. The solution can once again be found in the list of solutions to ODEs. 5.4.2 Definition of Transient and Steady State Response. If you have looked at the list of solutions to the equations of motion we derived in the preceding section, you will have discovered that they look horrible. Unless you have a great deal of experience with visualizing equations, it is extremely difficult to work out what the equations are telling A Java applet posted at should help to visualize the motion. The applet will open in a new window so you can see it and read the text at the same time. The applet simply calculates the solution to the equations of motion using the formulae given in the list of solutions, and plots graphs showing features of the motion. You can use the sliders to set various parameters in the system, including the type of forcing, its amplitude and frequency; spring constant, damping coefficient and mass; as well as the position and velocity of the mass at time t=0. Note that you can control the properties of the spring-mass system in two ways: you can either set values for k, m and using the sliders, or you can set , K and instead. We will use the applet to demonstrate a number of important features of forced vibrations, including the following: The steady state response of a forced, damped, spring mass system is independent of the initial conditions. To convince yourself of this, run the applet (click on `start' and let the system run for a while). Now, press `stop'; change the initial position of the mass, and press `start' again. You will see that, after a while, the solution with the new initial conditions is exactly the same as it was before. Change the type of forcing, and repeat this test. You can change the initial velocity too, if you wish. We call the behavior of the system as time gets very large the `steady state' response; and as you see, it is independent of the initial position and velocity of the mass. The behavior of the system while it is approaching the steady state is called the `transient' response. The transient response depends on everything... Now, reduce the damping coefficient and repeat the test. You will find that the system takes longer to reach steady state. Thus, the length of time to reach steady state depends on the properties of the system (and also the initial conditions). The observation that the system always settles to a steady state has two important consequences. Firstly, we rarely know the initial conditions for a real engineering system (who knows what the position and velocity of a bridge is at time t=0?) . Now we know this doesn't matter the response is not sensitive to the initial conditions. Secondly, if we aren't interested in the transient response, it turns out we can greatly simplify the horrible solutions to our equations of motion. When analyzing forced vibrations, we (almost) always neglect the transient response of the system, and calculate only the steady state behavior. If you look at the solutions to the equations of motion we calculated in the preceding sections, you will see that each solution has the form The term accounts for the transient response, and is always zero for large time. The second term gives the steady state response of the system. Following standard convention, we will list only the steady state solutions below. You should bear in mind, however, that the steady state is only part of the solution, and is only valid if the time is large enough that the transient term can be neglected. 5.4.3 Summary of Steady-State Response of Forced Spring Mass Systems. This section summarizes all the formulas you will need to solve problems involving forced vibrations. Solution for External Forcing Equation of Motion with Steady State Solution: The expressions for and are graphed below, as a function of (a) (b) Steady state vibration of a force spring-mass system (a) amplitude (b) phase. Solution for Base Excitation Equation of Motion with Steady State solution The expressions for and are graphed below, as a function of (a) (b) Steady state vibration of a base excited springmass system (a) Amplitude and (b) phase Solution for Rotor Excitation Equation of Motion with Steady State solution The expressions for and are graphed below, as a function of (a) (b) Steady state vibration of a rotor excited springmass system (a) Amplitude (b) Phase 5.4.4 Features of the Steady State Response of Spring Mass Systems to Forced Vibrations. Now, we will discuss the implications of the results in the preceding section. The steady state response is always harmonic, and has the same frequency as that of the forcing. To see this mathematically, note that in each case the solution has the form . Recall that defines the frequency of the force, the frequency of base excitation, or the rotor angular velocity. Thus, the frequency of vibration is determined by the forcing, not by the properties of the spring-mass system. This is unlike the free vibration response. You can also check this out using our applet. To switch off the transient solution, click on the checkbox labeled `show transient'. Then, try running the applet with different values for k, m and , as well as different forcing frequencies, to see what happens. As long as you have switched off the transient solution, the response will always be harmonic. The amplitude of vibration is strongly dependent on the frequency of excitation, and on the properties of the spring-mass system. To see this mathematically, note that the solution has the form . Observe that is the amplitude of vibration, and look at the preceding section to find out how the amplitude of vibration varies with frequency, the natural frequency of the system, the damping factor, and the amplitude of the forcing. The formulae for are quite complicated, but you will learn a great deal if you are able to sketch graphs of as a function of for various values of . You can also use our applet to study the influence of forcing frequency, the natural frequency of the system, and the damping coefficient. If you plot position-v-time curves, make sure you switch off the transient solution to show clearly the steady state behavior. Note also that if you click on the `amplitude-v- frequency' radio button just below the graphs, you will see a graph showing the steady state amplitude of vibration as a function of forcing frequency. The current frequency of excitation is marked as a square dot on the curve (if you don't see the square dot, it means the frequency of excitation is too high to fit on the scale if you lower the excitation frequency and press `start' again you should see the dot appear). You can change the properties of the spring mass system (or the natural frequency and damping coefficient) and draw new amplitude-v-frequency curves to see how the response of the system has changed. Try the following tests (i) Keeping the natural frequency fixed (or k and m fixed), plot amplitude-v-frequency graphs for various values of damping coefficient (or the dashpot coefficient). What happens to the maximum amplitude of vibration as damping is reduced? (ii) Keep the damping coefficient fixed at around 0.1. Plot graphs of amplitude-v-frequency for various values of the natural frequency of the system. How does the maximum vibration amplitude change as natural frequency is varied? What about the frequency at which the maximum occurs? (iii) Keep the dashpot coefficient fixed at a lowish value. Plot graphs of amplitude-v-frequency for various values of spring stiffness and mass. Can you reconcile the behavior you observe with the results of test (ii)? (iv) Try changing the type of forcing to base excitation and rotor excitation. Can you see any differences in the amplitude-v-frequency curves for different types of forcing? (v) Set the damping coefficient to a low value (below 0.1). Keep the natural frequency fixed. Run the program for different excitation frequencies. Watch what the system is doing. Observe the behavior when the excitation frequency coincides with the natural frequency of the system. Try this test for each type of excitation. If the forcing frequency is close to the natural frequency of the system, and the system is lightly damped, huge vibration amplitudes may occur. This phenomenon is known as resonance. If you ran the tests in the preceding section, you will have seen the system resonate. Note that the system resonates at a very similar frequency for each type of forcing. As a general rule, engineers try to avoid resonance like the plague. Resonance is bad vibrations, man. Large amplitude vibrations imply large forces; and large forces cause material failure. There are exceptions to this rule, of course. Musical instruments, for example, are supposed to resonate, so as to amplify sound. Musicians who play string, wind and brass instruments spend years training their lips or bowing arm to excite just the right vibration modes in their instruments to make them sound perfect. There is a phase lag between the forcing and the system response, which depends on the frequency of excitation and the properties of the spring-mass system. The response of the system is . Expressions for are given in the preceding section. Note that the phase lag is always negative. You can use the applet to examine the physical significance of the phase lag. Note that you can have the program plot a graph of phase-v-frequency for you, if you wish. It is rather unusual to be particularly interested in the phase of the vibration, so we will not discuss it in detail here. 5.4.5 Engineering implications of vibration behavior The solutions listed in the preceding sections give us general guidelines for engineering a system to avoid (or create!) vibrations. Preventing a system from vibrating: Suppose that we need to stop a structure or component from vibrating e.g. to stop a tall building from swaying. Structures are always deformable to some extent this is represented qualitatively by the spring in a spring-mass system. They always have mass this is represented by the mass of the block. Finally, the damper represents energy dissipation. Forces acting on a system generally fluctuate with time. They probably aren't perfectly harmonic, but they usually do have a fairly well defined frequency (visualize waves on the ocean, for example, or wind gusts. Many vibrations are man-made, in which case their frequency is known for example vehicles traveling on a road tend to induce vibrations with a frequency of about 2Hz, corresponding to the bounce of the car on its suspension). So how do we stop the system from vibrating? We know that its motion is given by Our vibration solution predicts that the mass vibrates with displacement Again, the graph is helpful to understand how the vibration amplitude varies with system parameters. Clearly, we can minimize the vibration amplitude of the mass by making . We can do this by making the spring stiffness as small as possible (use a soft spring), and making the mass large. It also helps to make the damping small. This is counter-intuitive people often think that the energy dissipated by the shock absorbers in their suspensions that makes them work. There are some disadvantages to making the damping too small, however. For one thing, if the system is lightly damped, and is disturbed somehow, the subsequent transient vibrations will take a very long time to die out. In addition, there is always a risk that the frequency of base excitation is lower than we expect if the system is lightly damped, a potentially damaging resonance may occur. Suspension design involves a bit more than simply minimizing the vibration of the mass, of course the car will handle poorly if the wheels begin to leave the ground. A very soft suspension generally has poor handling, so the engineers must trade off handling against vibration isolation. 5.4.6 Using Forced Vibration Response to Measure Properties of a System. We often measure the natural frequency and damping coefficient for a mode of vibration in a structure or component, by measuring the forced vibration response of the system. Here is how this is done. We find some way to apply a harmonic excitation to the system (base excitation might work; or you can apply a force using some kind of actuator, or you could deliberately mount an unbalanced rotor on the system). Then, we mount accelerometers on our system, and use them to measure the displacement of the structure, at the point where it is being excited, as a function of frequency. We then plot a graph, which usually looks something like the picture on the right. We read off the maximum response , and draw a horizontal line at amplitude . Finally, we measure the frequencies , and as shown in the picture. We define the bandwidth of the response as Like the logarithmic decrement, the bandwidth of the forced harmonic response is a measure of the damping in a system. It turns out that we can estimate the natural frequency of the system and its damping coefficient using the following formulae The formulae are accurate for small - say . To understand the origin of these formulae, recall that the amplitude of vibration due to external forcing is given by We can find the frequency at which the amplitude is a maximum by differentiating with respect to , setting the derivative equal to zero and solving the resulting equation for frequency. It turns out that the maximum amplitude occurs at a frequency For small , we see that Next, to get an expression relating the bandwidth to , we first calculate the frequencies and . Note that the maximum amplitude of vibration can be calculated by setting , which gives Now, at the two frequencies of interest, we know , so that and must be solutions of the equation Rearrange this equation to see that This is a quadratic equation for and has solutions Expand both expressions in a Taylor series about to see that so, finally, we confirm that 5.4.7 Example Problems in Forced Vibrations Example 1: A structure is idealized as a damped springmass system with stiffness 10 kN/m; mass 2Mg; and dashpot coefficient 2 kNs/m. It is subjected to a harmonic force of amplitude 500N at frequency 0.5Hz. Calculate the steady state amplitude of vibration. Start by calculating the properties of the system: Now, the list of solutions to forced vibration problems gives For the present problem: Substituting numbers into the expression for the vibration amplitude shows that Example 2: A car and its suspension system are idealized as a damped springmass system, with natural frequency 0.5Hz and damping coefficient 0.2. Suppose the car drives at speed V over a road with sinusoidal roughness. Assume the roughness wavelength is 10m, and its amplitude is 20cm. At what speed does the maximum amplitude of vibration occur, and what is the corresponding vibration amplitude? Let s denote the distance traveled by the car, and let L denote the wavelength of the roughness and H the roughness amplitude. Then, the height of the wheel above the mean road height may be expressed as Noting that , we have that i.e., the wheel oscillates vertically with harmonic motion, at frequency . Now, the suspension has been idealized as a springmass system subjected to base excitation. The steady state vibration is For light damping, the maximum amplitude of vibration occurs at around the natural frequency. Therefore, the critical speed follows from Note that K=1 for base excitation, so that the amplitude of vibration at is approximately Note that at this speed, the suspension system is making the vibration worse. The amplitude of the car's vibration is greater than the roughness of the road. Suspensions work best if they are excited at frequencies well above their resonant frequencies. Example 3: The suspension system discussed in the preceding problem has the following specifications. For the roadway described in the preceding section, the amplitude of vibration may not exceed 35cm at any speed. At 55 miles per hour, the amplitude of vibration must be less than 10cm. The car weighs 3000lb. Select values for the spring stiffness and the dashpot coefficient. We must first determine values for and that will satisfy the design specifications. To this end: (i) The specification requires that at resonance. Examine the graph of shown with the solutions to the equations of motion. Recall that K=1 for a base excited springmass system. Observe that, with , the amplification factor never exceeds 1.75. (ii) Now, the frequency of excitation at 55mph is We must choose system parameters so that, at this excitation frequency, . Examine the graph showing the response of a base excited springmass system again. We observe that, for , for . Therefore, we pick . Finally, we can compute properties of the system. We have that Similarly 85%(13)85% found this document useful (13 votes)8K viewsThis document contains example problems related to mechanical vibrations of single and multi degree-of-freedom systems. It includes solved problems on topics such as free and forced vibratio...AI-enhanced title and descriptionSaveSave Mechanical vibration solved examples For Later85%85% found this document useful, undefined This paper presents a series of mechanical vibration problems focused on single degree-of-freedom (DOF) systems, analyzing their free vibration characteristics, equations of motion, and responses under various conditions. Specific problems include determining spring constants, damping ratios, and steady-state amplitudes in mechanically complex configurations. Critical insights emphasize the importance of correctly modeling mechanical vibrations to understand system behavior under forced and unforced conditions. In this blog, we shall review the mathematical analysis of single degree of freedom lumped parameter systems with damping. The damping constant, c, is analogous to the spring constant, k, with the difference being that the force is proportional to velocity (whereas that force was proportional to displacement for a spring). If we consider the damped system in equilibrium as before and analyse the forces acting on the mass in a free body diagram (FBD) as shown below, it is possible to obtain the equation of motion via Newton's second law as shown. It can be seen that the equation of motion is similar to that for the undamped system, with the addition of an extra term to account for the damping now present in the model. If we now normalise with respect to mass (this means to divide through the equation by the mass) we obtain an alternative form of the equation of motion as shown below. This normalised version of the equation of motion can be used to infer the modal parameters of the system, namely the undamped natural frequency, ωn and the damping ratio, These modal parameters are distinct from the physical parameters of the system, and it is possible to describe the equation of motion either in physical parameters or modal parameters with no loss (or duplication) of information. Let us now consider the response of the system to various amounts of damping. The damping ratio,, can be either under damped, over damped or critically damped as shown below. Critical damping is the reference case, because here the damping in the system is such that there is no overshoot in the response, i.e. the system decays to zero without passing the equilibrium position. Typically, most engineering structures are lightly damped and have a damping ratio value in the single figures as a percentage. Here, there is overshoot followed by several oscillations before the system comes to rest. Damped Natural Frequency, ωd and Damped Time Period, Td The inclusion of damping in a system, has the effect of reducing the resonance frequency slightly which is said to occur at the damped natural frequency, ωd given by: ωd = ω n √1-2 Now, as is typically ~ 0.5 – 5%, then it is clear that for light damping: ωd ≈ ω n Note, that the time period is also slightly modified and is referred to as the damped time period, Td, : Td = 1/fd = 2π / ωd Finally, the time solution for the damped SDOF system can be summarised as follows: This occurs < 1 and c < cc. If we assume that t = 0 and the displacement, x = A at the moment the mass is released we get a decaying cosinusoidal oscillation as shown. The displacement is described by the following equation. x = Ae-ωnt cos(ωdt) A mass of 4 kg is suspended on a spring of stiffness 3000 N/m. The system is fitted with a damper with a damping ratio of 0.2. The mass is pulled down 50 mm and released. Calculate the following. a) the damped natural frequency. b) the displacement after 0.5 seconds Solution: a) ωd = √k/m = √3000/4 = 27.4 rad/s ωd = ω n √1-2 ωd = (27.4) √1- (0.2)2 = 26.8 rad/s b) Displacement: A=initial displacement = 25 mm = 0.025 m x = Ae-ωnt cos(ωdt) x = 0.025 e -(0.2)(27.4)(0.5) cos (26.8 x 0.5) x = 0.025 e -(2.74) cos (13.4) x = 0.025 (0.065)(0.67) = 1.08 X 10-3 m x = 1.08 mm A product with a mass of 2 kg is mounted on a spring which experiences a 0.5 mm extension when a force of 1.2 N is applied to it. A viscous damper system is also attached to the product which resists the motion with a force proportional to velocity equal to 25 N when the velocity is 5 m/s. a)Find the undamped natural frequency, the damping ratio and the damped natural frequency. b)Comment on the values found for the undamped and damped natural frequencies. c) Find the time period of the system d) state the equation of motion for the system Solution a) k = F/x = 1.2 / 0.5 x10-3 = 2400 N/m c = F / x . = 25 / 5 = 5 Ns/m Undamped natural frequency: ωd = √k/m = √2400/2 = 34.6 rad/s Critical damping constant is: cc = 2ωn m cc = 2(34.6)(2) = 138.6 Ns/m Damping ratio is: c = 2ωn m = c / 2ωn m = c/cc = 5 /138.6 = 0.036 = 3.6% Damped natural frequency: ωd = ω n √1-2 ωd = (34.6) √1-(0.036)2 ωd = (34.6)(0.999) ≈ 34.6 rad/s b) The undamped and damped natural frequencies are very similar. The reason is due to the value of √1-2 which will take the value of √≈1 ≈1 when the value of the damping ratio,